Peak Value This is the maximum value V_M or I_M. For example, specifying that a sine wave has a peak value of 170 V states how much it is, since all other values during the cycle follow a sine wave. The peak value applies to either the positive or the negative peak.

In order to include both peak amplitudes, the *peak-to-peak* (p-p) *value* may be specified. For the same example, the peak-to-peak value is 340 V, double the peak value of 170 V, since the positive and negative peaks are symmetrical. It should be noted, though, that the two opposite peak values cannot occur at the same time. Furthermore, in some waveforms the two peaks are not equal.

Average Value This is an arithmetic average of all the values in a sine wave for one alternation, or half-cycle. The half-cycle is used for the average because over a full cycle the average value is zero, which is useless for comparison purposes. If the sine values for all angles up to 180°, for one alternation, are added and then divided by the number of values, this average equals 0.637. These calculations are shown in Table 16-2.

Since the peak value of the sine is 1 and the average equals 0.637, then

$$\text{Average value} = 0.637 \times \text{peak value} \qquad \textbf{(16-2)}$$

With a peak of 170 V, for example, the average value is 0.637 × 170 V, which equals approximately 108 V.

Root-Mean-Square, or Effective, Value The most common method of specifying the amount of a sine wave of voltage or current is by stating its value at 45°, which is 70.7 percent of the peak. This is its *root-mean-square* value, abbreviated rms. Therefore,

$$\text{rms value} = 0.707 \times \text{peak value} \qquad \textbf{(16-3)}$$

or

$$V_{rms} = 0.707V_{max} \qquad \text{and} \qquad I_{rms} = 0.707I_{max}$$

With a peak of 170 V, for example, the rms value is 0.707 × 170, or 120 V, approximately. This is the voltage of the commercial ac power line, which is always given in rms value.

It is often necessary to convert from rms to peak value. This can be done by inverting Formula (16-3) as follows:

Table 16-2. Derivation of Average and RMS Values for a Sine-Wave Alternation

Interval	Angle θ	Sin θ	(Sin $\theta)^2$
1	15°	0.26	0.07
2	30°	0.50	0.25
3	45°	0.71	0.50
4	60°	0.87	0.75
5	75°	0.97	0.93
6	90°	1.00	1.00
7*	105°	0.97	0.93
8	120°	0.87	0.75
9	135°	0.71	0.50
10	150°	0.50	0.25
11	165°	0.26	0.07
12	180°	0.00	0.00
	Total	7.62	6.00
	Average →	$\frac{7.62}{12} = 0.635$†	$\sqrt{\frac{6}{12}} = \sqrt{0.5} = 0.707$

*For angles between 90 and 180°, sin θ = sin (180° − θ).
†More intervals and precise values are needed to get the exact average of 0.637.

$$\text{Peak} = \frac{1}{0.707} \times \text{rms} = 1.414 \times \text{rms} \qquad (16\text{-}4)$$

or

$$V_{max} = 1.414V_{rms} \qquad \text{and} \qquad I_{max} = 1.414I_{rms}$$

Dividing by 0.707 is the same as multiplying by 1.414.

For example, the commercial power-line voltage with an rms value of 120 V has a peak value of 120 × 1.414, which equals 170 V, approximately. Its peak-to-peak value is 2 × 170, or 340 V, which is double the peak value. As a formula,

$$\text{Peak-to-peak value} = 2.828 \times \text{rms value} \qquad (16\text{-}5)$$

The factor 0.707 for rms value is derived as the square root of the average (mean) of all the squares of the sine values. If we take the sine for each angle in the cycle, square each value, add all the squares, divide by the number of values added to obtain the average square, and then take the square root of this mean value, the answer is 0.707. These calculations are shown in Table 16-2 for one alternation from 0 to 180°. The results are the same for the opposite alternation.

The advantage of the rms value derived in terms of the squares of the voltage or current values is that it provides a measure based on the ability of the sine wave to produce power, which is I^2R or V^2/R. As a result, the rms value of an alternating sine wave corresponds to the same amount of direct current or voltage in heating power. An alternating voltage with an rms value of 120 V, for instance, is just as effective in heating the filament of a light bulb as 120 V from a steady dc voltage source. For this reason, the rms value is also the *effective* value.

Unless indicated otherwise, all sine-wave ac measurements are in rms values. The capital letters V and I are used, corresponding to the symbols for dc values. As an example, V = 120 V for the ac power-line voltage.

The ratio of the rms to average values is the *form factor*. For a sine wave, this ratio is 0.707/0.637 = 1.11.

Note that sine waves can have different amplitudes but still follow the sinusoidal waveform. Figure 16-7 compares a low-amplitude voltage with a high-amplitude voltage. Although different in amplitude, they are both sine waves. In each wave, the rms value is 0.707 of the peak value.

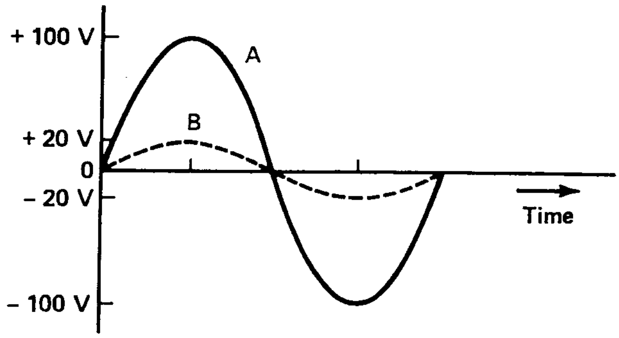

Fig. 16-7 Waveforms A and B have different amplitudes, but both are sine waves.

Practice Problems 16-5
Answers at End of Chapter

a. Convert 170 V peak to rms value.
b. Convert 10 V rms to peak value.

16-6 Frequency

The number of cycles per second is the *frequency*, with the symbol f. In Fig. 16-3, if the loop rotates through 60 complete revolutions, or cycles, during 1 s, the frequency of the generated voltage is 60 cps, or 60 Hz. You see only one cycle of the sine waveform, instead of 60 cycles, because the time interval shown here is 1⁄60 s. Note that the factor of time is involved. More cycles per second means a higher frequency and less time for one cycle, as illustrated in Fig. 16-8. Then the changes in values are faster for higher frequencies.

A complete cycle is measured between two successive points that have the same value and direction. In Fig. 16-8 the cycle is between successive points where the waveform is zero and ready to increase in the positive direction. Or the cycle can be measured between successive peaks.

On the time scale of 1 s, waveform a goes through one cycle; waveform b has much faster variations, with four complete cycles during 1 s. Both waveforms are sine waves, even though each has a different frequency.