SELF-RESONANCE IN COILS
and the self-capacitance myth

All coils show a self-resonant frequency (SRF), and as this frequency is approached the inductance and resistance increase while the Q decreases until a frequency is reached where the coil resonates in a similar way to a parallel tuned circuit. Because of this similarity the effect has been attributed to self-capacitance in the coil, and many researchers have tried to reduce this capacitance in order to raise the Q. However nowhere in the coil can this capacitance be measured or deduced, and in fact the rising inductance and loss are explained if the coil is seen as a helical transmission line. This article discusses these issues, and gives accurate equations for the changes with frequency.

1. INTRODUCTION

All coils show a self-resonant frequency (SRF) and as this frequency is approached the inductance and resistance increase while the Q decreases. This effect exactly mirrors a parallel tuned circuit, and because of this the accepted theory assumes that the coil has self-capacitance which along with its inductance produces this resonance. So in this theory the inductance is *constant* with frequency and the changes seen are due to the effects of a parallel self-capacitance. Further support for the self-capacitance theory is given by the fact that the measured changes are accurately modeled by a parallel tuned circuit. However, nowhere in the coil can this capacitance be measured or deduced, and this is because this capacitance does not exist and the resonance is due to standing waves on the wire, similar to a transmission-line (see Payne ref 1). The inductance change is therefore a real change and not an apparent change caused by a fictitious self-capacitance. [There probably *is* some self-capacitance, and this can be significant in multilayer coils, but in single layer coils it is not significant].

So given the more likely transmission-line mechanism why do the equations based upon a parallel circuit produce such accurate results? This reason is that the change in reactance with frequency of the transmission-line mode follows very closely that of a parallel tuned circuit, and differs by less than 5% for frequencies up to 80% of the SRF. The self-capacitance equations therefore give a very useful model for predicting the change in inductance with frequency but have encouraged researchers in a futile search for this capacitance and its reduction. Different winding techniques have been tried such as basket weaving, but the gains made here (if any) are likely to be due to a reduction in the dielectric loss in the winding former, or more likely in the raising of the self-resonant frequency of the coil by a change its transmission line phase velocity.

In writing this article the author needed to address a presentational problem. This arises because it is accepted practice to view the inductance of a coil as *independent of frequency* and the changes seen as being apparent changes due to a fictitious self-capacitance. This article shows that the changes are in fact real, but equally that the self-capacitance model provides a very good means of calculating these changes. The problem arises when a coil is measured and compared with theory – should the measurements be corrected for the SRF to identify the underlying 'constant inductance', or should the rising inductance be taken as the truth and the SRF effect be included in the *theory*?. The latter is the strictly correct approach, but for validating the theory it doesn't matter which approach is taken. Given that it is common practice to adjust the measurements for the effects of SRF, that is the approach here.

The theoretical analysis in this article has been supported by measurements on the coil described in Section 6 (Measurements).

Key equations are highlighted in red.

2. THE EFFECT OF SRF ON INDUCTANCE, RESISTANCE AND Q

If a coil with a parallel capacitance is used in a parallel tuned circuit the effect of this capacitance (whether self-capacitance or added capacitance) is merely to reduce the capacitance necessary to resonate at any particular frequency. But if the coil is used in a series tuned circuit the effect of the parallel capacitance is to reduce the Q, increase the *apparent* inductance and to increase the *apparent* series resistance. Welsby (ref 2) derives the following equations, where Q, L and R, are values which would obtain in the absence of the SRF and Q_m, L_m and R_m are the values that would be measured at a frequency f, for a self-resonant frequency of f_r :

$$Q = Q_m / [\, 1- (f / f_r)^2] \qquad 2.2.1$$

$$L = L_m [\, 1- (f / f_r)^2] \qquad 2.2.2$$

$$R = R_m \,[\, 1- (f / f_r)^2]^2 \qquad 2.2.3$$

These equations are valid for series inductance if the Q is high enough for $(1+1/Q^2)$ to be taken as unity ie for Q >4.

These equations can be used to remove the effects of the SRF from measurements or alternatively (by transposition) to be included in the theoretical equations so that these then include the effects of the SRF.

Transposing Equation 2.2.1 to $Q_m = Q\,[\,1- (f / f_r)^2]$ shows why it is desirable for the SRF to be as high as possible because as this frequency is approached the measured Q decreases, tending to zero at the SRF (see also Section 3).

To assess the accuracy of these equations a comparison of the measured and predicted inductance of the test coil is given below:

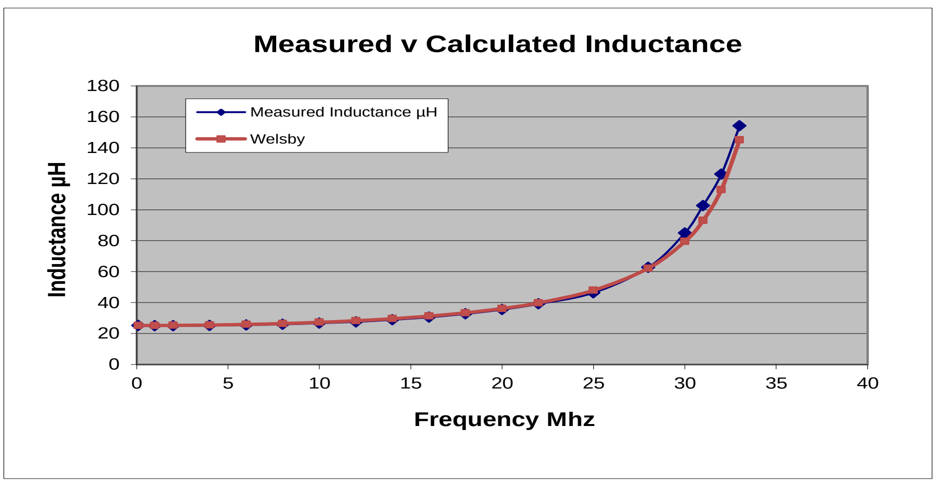

Figure 2.1 Measured and Calculated Inductance

For the calculated inductance an SRF of 36.3 MHz was used, calculated from Equation 5.6.2. The agreement is within 6 % for all frequencies up to 30 MHz, or up to 83% of the SRF. The accuracy is improved to better than 5% over the whole range shown if the SRF is assumed to be slightly lower at 36 MHz.

To evaluate the accuracy of Equation 2.2.3 for resistance is a little more complicated, because the underlying resistance R itself varies with frequency due to skin effect and proximity effect. This is analysed in Appendix 1, and the following equation derived :

$$\mathbf{R_m = K \sqrt{f} \; / \; [\, 1 - (f / f_r)^2]^2} \qquad \textbf{2.2.4}$$

$$\textbf{Where } \mathbf{K = R_{dc} \;\; \varphi \;\; 0.25 \; d_w/66.6}$$

A comparison with experiment is shown below for the test coil :

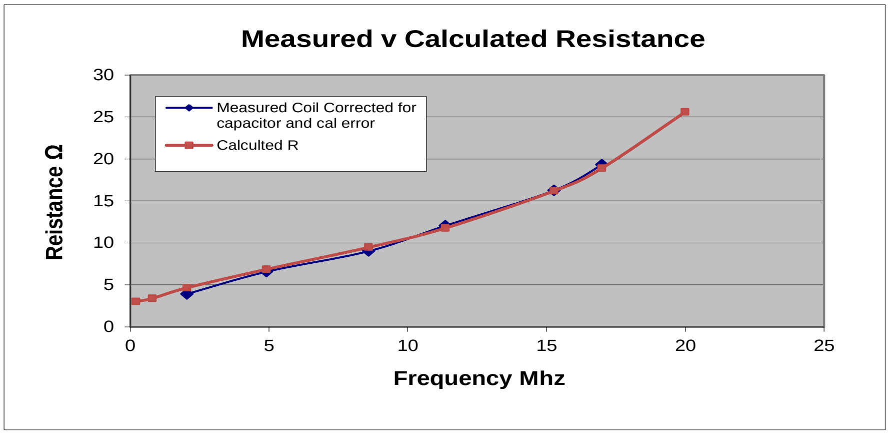

Figure 2.2 Measured and Calculated Resistance

For the measurements, the coil reactance was tuned-out with an air variable capacitor and its resistance subtracted from the measurements (see Section 6).

3. THE EFFECT OF SRF ON COIL Q

A typical air coil wound with solid wire has a resistance which increases as the square root of frequency (for $d_w/\delta >> 1$) and this is due to the skin effect and the proximity effect. In the absence of self-resonance the reactance of a coil would increase in direct proportion to frequency so we could expect the Q to increase as $\sqrt{f}$, but in practice the Q rises with frequency up to a flat maximum and then decreases at higher frequencies. This reduction of the Q at high frequencies is often attributed to losses in the dielectric supports, or the covering of the wire such as the enamel, however the main mechanism is self-resonance, as shown below.

Appendix 1 shows that the overall resistance varies with frequency as :

$$R_m = K \sqrt{f} \; / \; [\, 1 - (f / f_r)^2]^2 \qquad 3.1$$

The coil Q is the ratio of the coil reactance $X_{Lm} = 2\pi f \, L / ([\, 1 - (f / f_r)^2]$ to the resistance given by the above equation. Collecting the factors which are independent of frequency and putting them equal to K', this gives :

$$\mathbf{Q_m = K' \, f^{\,0.5} \, [1 - (f/f_r)^2]} \qquad \textbf{3.2}$$

$$\textbf{Where } \mathbf{K' = 2\pi L \; R_{dc} \;\; \varphi \;\; 0.25 \; d_w/66.6}$$

This is plotted below with the maximum value normalised to unity, and frequency normalised to the SRF. This curve closely follows the change seen with practical coils.

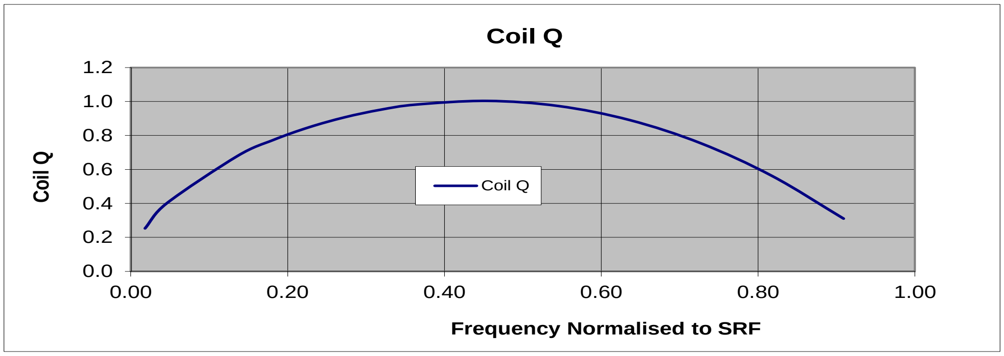

Figure 3.1 Coil Q with the effects of SRF

It is important therefore to be able to calculate the SRF, and two methods are described below, firstly Medhurt's empirical self-capacitance model and secondly a theoretical transmission-line model.

4. SRF FROM THE SELF-CAPACITANCE MODEL

The most extensive work on the capacitive approach is in a paper by Medhurst (ref 3), who measured a number of coils and derived an empirical equation for their parallel self-capacitance which matched his measurements :

$$C_o = 0.1126\, l_c + 0.08\, d_c + 0.27(d_c{}^3 / l)^{0.5} \quad \text{pf} \qquad 4.1.1$$

where l_c and d_c are the coil length and diameter in cm.

For the test coil described in Section 6 the capacitance according to the above equation is 0.58 pf. Notice how very small this is, and so even a small stray capacitance of the test jig or the leads to the coil can affect the value considerably, and therefore the SRF.
Taking the usual equation for the resonant frequency of a lossless parallel resonant circuit, the SRF is :

$$f_{srf} = 1/[2\pi\, (LC_o)^{0.5}] \qquad 4.1.2$$

Notice that the capacitance Equation 4.1.1 is dependent only on the overall dimensions of the coil and independent of the number of turns, the pitch of the winding or the diameter of the conductor. However these factors do affect the SRF and appear in Equation 4.1.2 in the calculation of the inductance.

For the test coil, 0.58pf resonates with its inductance of 25.3µH at f_r = 41.4 MHz. For comparison the SRF was measured as 36.4 MHz (see Section 6), 14% less than that given by the above two equations.
Attempts have been made by a number of researchers to produce a *theoretical* basis for this capacitance, the most notable being by Polermo (ref 4). He produced an equation based on the concept of inter-turn capacitance, which was supported by his own experiments. However his theory was demolished by Medhurst who came very close to accusing Polermo of adjusting his results to fit his theory.

5. SRF FROM THE TRANSMISSION-LINE MODEL

5.1. Introduction

Lumped electronic components such as resistors and capacitors can be considered as having values which are independent of frequency. This is a reasonable assumption up to around 100 MHz, but above this frequency the component must be modified for so called 'strays' or 'parasitic effects' such as a small inductance in series with the capacitor or a small capacitance across the resistor. At frequencies above 1 GHz normal lumped components must now be considered as transmission lines, and indeed even at lower frequencies in that the strays are actually approximations to the underlying transmission line. So a capacitor, for instance, is in reality a section of open-circuited transmission line, and at high frequencies when the length of the capacitor approaches a wavelength additional lumped components must be added to give a closer approximation to the transmission line reactance.
Inductors were intentionally not mentioned in the above because they are hardly ever truly 'lumped', in that the length of wire needed to obtain a useful inductance at any frequency can never be truly small compared to the operating wavelength. So the transmission line mode on the wire is never far above the useful operating frequencies (see Payne ref 1). A coil grounded at one end must be considered therefore as a helical transmission line shorted at one end. As with all shorted lines if the frequency is sufficiently low the input impedance approximates to a series fixed inductance. This is the classic low frequency inductance of a coil given by the familiar equation :

$$\mathbf{L = \mu_o N^2 a_c^2 K_n \ / \ \ell_c} \qquad \mathbf{5.1.1a}$$

where a_c and ℓ_c are the radius and length of the coil

The factor K_n is known as Nagaoka's factor and is given approximately by (Welsby ref 2):

$$\mathbf{K_n \approx 1 \ / \ [1{+}0.45\ d_c / \ell_c - 0.005\ (d_c / \ell_c)^2]} \qquad \mathbf{5.1.1b}$$

where d_c is the coil diameter

When a coil is used at frequencies where its inductive reactance is useful the inductance will be somewhat greater than that calculated from Equation 5.1.1, and at a sufficiently high frequency the reactance will become very high (infinite if there were no losses), and this frequency is known as the Self Resonant Frequency (SRF). As an example a coil useful at say 1MHz will probably have an SRF around 20 MHz. A more extreme example is a loading coil for a whip antenna, and the loading coils described in Radio Amateur handbooks show that the length of wire at say 3.8 MHz, is close to $\lambda/4$ at this frequency.
So the inductance at any frequency can be found from the two Equations 5.1.1 and 2.2.2, as long as the SRF is known, and this is dependent on the phase velocity down the helical line, the end effect, and the dielectric constant of the winding former. These are considered below.

5.2. Phase Velocity

In the past experimenters have attempted to raise the SRF of a coil by reducing its self-capacitance, but no such capacitance exists and attention must instead be concentrated on increasing the phase velocity down the helical transmission line.
Initially considering a two wire transmission line (with air between the lines) or a straight single wire such as an antenna, EM waves travel down them with a velocity very close to that of the speed of light, c,. If the straight wire is made into a helix the wave follows the helical path along the wire at the speed of light *if* the diameter of the helix is comparable to a wavelength or larger. The SRF can then be calculated knowing the length of the wire.
When the diameter is small compared to a wavelength (the normal condition) the wave again follows a helical path but this time with a pitch somewhat larger than that of the wire, up to 4 times larger or more. One way to look at this is that the phase velocity *down the wire* exceeds that of the velocity of light c, and so the wire appears shorter and the SRF is raised.

Kandoian & Sichak (ref 5) have studied this theoretically and have shown that the velocity down the wire, relative to the speed of light, V_w/c, depends upon the pitch of the winding, its diameter and the frequency of operation. Their equation can be written as:

$$V_w/c = V_w' = [(1+ x^2)/ (1+(k\,x)^2)]^{0.5} \qquad 5.2.1$$

where $x = 2\pi\, a/p$

$k = \sqrt{20/\pi}\, [d_c^{\,2} f/(300\, p)]^{0.25}$

d_c is the coil diameter in metres

p is the winding pitch in metres

f is in Mhz

Notice that the numerator gives the path length of the wire (per meter of coil) and the denominator that of the wave. So the velocity is dependent upon the coil radius, a, the winding pitch p and the frequency. For the test coil if the pitch is changed the relative phase velocity for f=20 MHz is given in blue below :

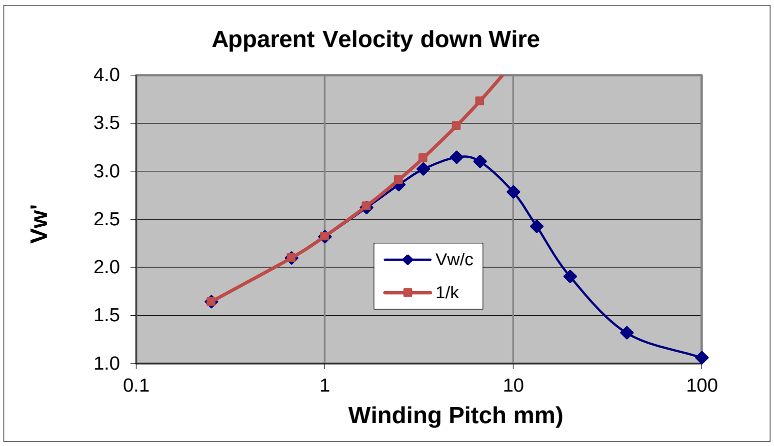

Figure 5.2.1 Relative Velocity down Wire

Also shown (in brown) is the approximation 1/k.
When the pitch is very small or very large the relative **v**elocity is close to unity (ie close to the velocity of light). There is a pitch where the velocity is a maximum, and so for a given length of wire this will give the highest SRF (ignoring end-effect see later). This pitch is around 5.5 mm or about equal to the coil radius in this particular case, but it is found that this is general and a pitch equal to the radius gives close to the maximum velocity. However a coil with such a large pitch is not very useful as an inductor.
If $(kx)^2 >> 1$ then Equation 4.2.1 simplifies to $V_w' \approx 1/k$, so :

$$\mathbf{V_w' \approx 1/\,k \;=\; [73\, p/ (d_c^{\,2} f)]^{0.25}} \qquad \mathbf{5.2.2}$$

Generally for normal coils in the HF frequency range this approximation applies when the coil diameter is more than 4.5 times the pitch ($d_c/p > 4.5$), which is normally the case, to give accuracy generally better than 10%.

5.3. λ/4 and λ/2 Modes

With the wire acting as a transmission-line it will resonate when the wire length corresponds approximately to nλ/4, where n is an integer 1,2,3 etc.
Normally one end of the coil is grounded and experiments show that this normally leads to the λ/4 mode, with the grounded terminal providing an effective transmission-line short circuit. This is also the mode when the coil is inserted into a test jig for the measurement of inductance, resistance and SRF.
The λ/2 mode can also be excited. For this the coil is suspended away from other objects and ground, and energy coupled into it via a coupling loop of a few turns, located at the centre of the coil. The whole

arrangement is now balanced against ground. Resonance in the coil can be determined by measuring the input impedance of the loop, and this will peak at the SRF.

5.4. End Effect

In addition to the phase velocity there is another factor affecting the SRF and this is the end-effect, whereby the coil seems to be longer than its physical length because its fields extend beyond its ends.

Given that the coil is like a transmission line an insight into end effect can be obtained by considering a two wire transmission line, short-circuited at the far end. Its first resonance will be when its length approximately equal to $\lambda_g/4$ and then it will resonate like a parallel LC circuit (λ_g is the wavelength of wave propagation on this line). Indeed at VHF and UHF $\lambda/4$ transmission lines are often used in the place of conventional LC circuits.

In the above care was taken to say that the line length is only *approximately* $\lambda g/4$ for resonance, and this is because of the 'end effect', whereby the transmission line appears to be longer than its physical length, by up to 15% of even more. Of course the coil is not a two wire transmission line since it has only one conductor, but standing waves can also appear on single wires, and the quarter wave monopole antenna and the half wave dipole are good examples. It has been found that the end effect appears here also, and can be calculated from the capacitance of the end disc formed from the cross-section of the conductor, along with a capacitance due to charge accumulation at the end of the line. With conventional wire antennas where the conductor diameter is small, this capacitance is very small, and so the end effect is around only 2%. However if the antenna is short and fat the end effect can be very large, and for instance Schelkunoff & Friis (ref 6) shows a curve where the end effect is 15% when the antenna diameter is equal to $\lambda/50$. One might think that if the end of a fat antenna was hollowed out to form a tube the end capacitance would reduce noticeably and with it the end effect. However early workers in this area were surprised to find 'not the slightest difference….' (Brown and Woodward ref 7).

And this brings us to the helix, which of course is also hollow. Measurements by the author indicate that when the helix dimensions are much less than a wavelength the end effect is of the order of the radius of the helix. This conclusion can also be derived from Equation 5.1.1, where it can be seen that the coil length ℓ_c is extended by $[1+0.45\ d_c / \ell_c - 0.005\ (d_c / \ell_c)^2]$. If the coil is not too short {ie $\ell_c > d_c/5$}, this can be approximated to $(1+0.45\ d_c / \ell_c)$, so that the length ℓ_c is extended to $\ell_c\ (1+0.45\ d_c)$. That is, the helix seems to be extended by $0.45\ d_c$ or $0.225\ d_c$ at each end.

The apparent length of the *wire* ℓ_w increases by the same factor, so that $\ell_w' = \ell_w\ (1+\delta)$, where ℓ_w is the physical wire length and δ is the end effect. Notice that it is the coil diameter and length which determine the extension of the wire and not the wire diameter and length.

The above applies to the $\lambda/2$ mode where both ends of the wire are free and then $\delta = [0.45\ d_c / \ell_c]$. But in the $\lambda/4$ mode one end is grounded, and there is then no wire extension at that end. So for the $\lambda/4$ mode the extension is $\delta/2$.

As an example the test coil of Section 6, having d_c = 11.4mm and ℓ_c = 22mm, the end effect is 0.23 (ie 23%) and 0.12 (12%) respectively. Notice how large these are, and these are the amounts by which the SRF is *reduced* due to end effect.

5.5. The Effect of Dielectric on SRF

The permittivity of the former on which the coil is wound will reduce the phase velocity, and thereby reduce the SRF. However, the electric field inside a coil is very small, so displacement currents are very small and the effect of the dielectric is therefore much reduced. Support for this view comes from Sichak (ref 8) who has analysed a coaxial cable with a helical inner line, and who says 'The significant parameter is $(2\pi a/N)(2\pi a/\lambda)$, where N= number of turns per unit length, a=radius and λ=wavelength. When this parameter is considerably less than 1, the velocity and characteristic impedance depend only on the dimensions. The *dielectric inside the helix has only a second order effect,*………' (my italics).

The Sichak criterion $F_{Sichak} = (2\pi aN)(2\pi a/\lambda)$, is more conveniently expressed as $(2\pi a/p)(2\pi a/\lambda)$, where p is the pitch of the winding. Generally for HF inductance coils, this has a value of less than unity but not necessarily considerably less than unity. For instance the test coil described later gives a value of F_{Sichak} = 0.3 at 20 MHz but the much larger diameter coil such as used for antenna loading may have a value F_{Sichak} = 0.7 at 20 MHz. Unfortunately Sichak does not give the values of the apparent dielectric constant for this range of values, but based upon Sichak's paper Payne (ref 9) has derived the following approximate equation for a dielectric which totally fills the inside of the coil :

$$\varepsilon' \approx [(\varepsilon'_r - 1)(F_{Sichak})^{1.5}]/8 + 1 \qquad 5.5.1a$$

where $F_{ishcak} = (2\pi a/p)(2\pi a/\lambda)$

ε'_r is the apparent dielectric constant of the former (see below)

The above equation is valid for values of F_{Sichak} up to 1.25.
Normally the winding former is a hollow tube and therefore does not fill the coil, and this will reduce the apparent dielectric constant of the material ε_r . There is no known equation for this reduction but it is assumed here that the electric susceptibility (ε_r -1) reduces in proportion to the area of the dielectric to the area of the coil. If the thickness of the dielectric is t, and this is small compared with the coil diameter d_c, then:

$$\varepsilon'_r \approx (\varepsilon_r - 1)\, 4t/d_c + 1 \qquad 5.5.1b$$

where ε_r is the relative dielectric constant of the material

As an example, a tubular former with a material dielectric constant of 2.6 and a thickness of 0.1 d_c will have $\varepsilon'_r = 1.64$. Using this value in Equation 5.5.1a for $F_{ishcak} = 0.3$ (the test coil) gives ε' of 1.013 to give a reduction in phase velocity of √1.013 = 1.006, or a 0.6% change in SRF. A large loading coil with F_{ishcak} = 0.7 would have a more significant 2.4% reduction in SRF, and in these coils it is common practice to reduce the former to no more than thin supporting strips.
The effect on the SRF of a tubular dielectric former is therefore very small, unless the factor (2πa/p)(2πa/λ) approaches unity *and* the dielectric constant is very high and this is unlikely in single layer inductance coils.

5.6. SRF from Transmission-Line Model

Combining the equations for end effect and velocity these give for *half wave* resonance:

$$\mathbf{f_{SRF} \approx [300*0.5 / \ell_w']^{0.8} / [d_c^{\,2}/ (73\,p)]^{0.2} \quad MHz \qquad 5.6.1}$$

where $\ell_w' = \ell_w (1+ 0.45\, d_c / \ell_c)$
ℓ_w is the length of the wire
d_c and ℓ_c are the diameter and length of the coil

And for *quarter wave* resonance :

$$\mathbf{f_{SRF} \approx [300*0.25 / \ell_w']^{0.8} / [d_c^{\,2}/ (73\,p)]^{0.2} \quad MHz \qquad 5.6.2}$$

where $\ell_w' = \ell_w (1+ 0.225\, d_c / \ell_c)$

If the effective dielectric constant of the winding former is significant, then the above frequencies should be divided by √ ε', given by Equation 5.5.1 a & b.

5.7. Comparison with Experiment and Errors

For the test coil the SRF calculated from Equation 5.6.2 is 36.3 MHz (λ/4), compared with the measured value of 36.4 MHz, a difference of only 0.2%. This accuracy would seem to be fortuitous, except that a check against an independent source gives a similar level of accuracy. Knight (ref 10) measured the λ/2 resonance of a very much larger coil, using a measurement set-up which went to great lengths to minimalise any loading from the measurement apparatus. His coil had a mean diameter (to the centre of the wire) of 96 mm, a length of 152 mm, a winding pitch of 8.4 mm and a wire length of 5.458 m. He measured the λ/2 SRF at 26.69 MHz and Equation 3.5.1 gives 26.85 MHz, a difference of only 0.6%.
The accuracy is surprising given that there is uncertainty in the diameter at which the current flows, and it may be significant that in both calculations the mean diameter to the centre of the wire was used. The good accuracy suggests that this is indeed the diameter of the current flow, at least at the resonant frequency.

This conclusion is consistent with Payne (ref 1) who showed that as the SRF is approached, that part of the total inductance due to the wire transmission line dominates and that part due to the normal 'coil mode' is suppressed. It is this coil mode which gives the strong central flux which causes the current to concentrate in the wire towards the axis of the coil, and this leads to an uncertainty in the diameter of current flow.

5.8. Summary

The transmission-line model has two main aspects : an end-effect which *reduces* the SRF, and an increased phase velocity which *increases* the SRF. For some dimensions of coil and winding-pitch these two effects can be equal and cancel, so that the length of the wire will be exactly equal to $\lambda_0/4$ or $\lambda_0/2$, where λ_0 is the free space wavelength. The velocity down the wire then *appears* to be equal to c, and interestingly Knight's coil (ref 10) has this characteristic, presumably by accident.

6. MEASUREMENT METHOD

6.1. Test Coil

To test the equations here a coil was wound with 75 turns of enamelled wire of copper diameter 0.234 mm. The coil length (*lc*) was 22mm, and it was wound onto a tubular plastic former 0.5 mm thick with a mean diameter to the centre of the wire, d_c, of 11.4 mm. The ratio of the wire diameter to pitch, d_w/p, was therefore 0.8. The wire length was 2.69 m, and the measured low frequency inductance was 25.3 μH (measured at 0.2 MHz).

6.2. Measurement of SRF

The SRF of the test coil was measured as 36.4 MHz, and this was done by earthing one end of the coil via a short lead, leaving the other end open. Coupling into the network analyser (VNA) was via a two turn loop located at the earthed end of the coil.

In carrying out the measurements it was very important that there was no lead at the open end of the coil, as even a short length of 0.23 mm dia wire (50 mm long) would reduce the SRF by 15% to 31 MHz. This lower frequency was also that measured when the coil ends were connected directly to a VNA via 50 mm leads, and so the capacitance of the leads is significant (as Medhurst discovered in his measurements).

6.3. Measurement of Inductance and Resistance

Measurements were made with an Array Solutions UHF analyser, with the impedance of the connection leads calibrated out. The resistance measurements were subject to large uncertainties because of the presence of the very high inductive reactance and so this reactance was tuned out with a high quality variable capacitor. This had silver plated vanes and wipers, and ceramic insulation and had a resistance given by (see Payne ref 11) :

$$R_{cap} = 0.01 + 800/(f\,C^2) + 0.01\,f^{0.5} \qquad 6.3.1$$

Where C is in pf, and f in MHz

This resistance was used to correct the measured results but it was always small compared with the overall measured resistance, and never more than 10%.

7. SUMMARY

Equations 5.6.1 and 5.6.2 give the SRF of a coil to a high accuracy for the λ/2 and λ/4 modes respectively. This SRF value can then be used in Equations 2.2.1, 2.2.2 and 2.2.3 to predict the change in the inductance, resistance, and Q due to self-resonance.

8. APPENDIX 1 : Resistance Change with Frequency

In Equation 2.2.3 the underlying resistance R is itself frequency dependent because of skin effect and proximity effect. The loss in a coil can be written in terms of the dc resistance of the wire R_{dc} :

$$R = R_{dc} \ \varphi \ H \qquad 8.1$$

Where $R_{dc} = 4 \rho \ell / (\pi d^2_w)$
ρ = resistivity ($1.77\ 10^{-8}$ for copper)
$H \approx 0.25 \ d^2_w / (d_w \delta - \delta^2)]$ (for $d_w / \delta > 1.6$)
δ is the skin depth, for copper = $66.6/\sqrt{f}$ mm
d_w is the diameter of the wire

Inserting this into Equation 2.2.3 gives the measured resistance as :

$$R_m = [R_{dc} \ \varphi \ H] / [1 - (f / f_r)^2]^2 \qquad 8.2$$

H is a multiplier due to the skin effect (see Payne ref 1) and φ is a multiplier which accounts for the increase in resistance due to the magnetic field from adjacent turns, the proximity effect. Payne (ref 12) gives an accurate equation, and this can be approximated to :

$$\varphi = 1 + k_r K_n^2 + 16 \pi (1 - K_n) (d_w / p)^2 M^2 \qquad 8.3$$

where $k_r \approx 2.6 (d_w / p) - 0.65$
$K_n \approx 1 / [1 + 0.45 (d_{coil} / l_{coil})]$
$M \approx d_{coil} / [(2 d_{coil})^2 + (l_{coil})^2]^{0.5}$

The above equation is accurate to within ±6% for N>15 turns, $(l_{coil} / d_{coil}) > 1.6$, and $(d_w / p) > 0.35$.
For the test coil described earlier, N= 75, $(l_{coil} / d_{coil}) = 1.9$, and $(d_w / p) = 0.82$ and the above equation gives $\varphi = 2.85$ (for comparison, Medhurt's closest tabulated measurement for $(l_{coil} / d_{coil}) = 2$, and $(d_w / p) = 0.8$ is 2.74, a difference of 4%).
At high frequencies, such that $d_w \delta >> 1$, H becomes $H \approx 0.25 \ d_w / \delta$, so Equation 8.2 reduces to :

$$R_m = [R_{dc} \ \varphi \ 0.25 \ d_w \sqrt{f} / 66.6] / [1 - (f / f_r)^2]^2 \qquad 8.4$$

Grouping the terms which are independent of frequency and putting these equal to K, this gives :

$$R_m = K \sqrt{f} / [1 - (f / f_r)^2]^2 \qquad 8.5$$

Where $K = R_{dc} \ \varphi \ 0.25 \ d_w / 66.6$

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