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Toward a more complete electrodynamic theory

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Abstract: Maxwell's equations require a gauge condition for specific solutions. This incompleteness motivates use of a dynamical quantity, $\xi = -\nabla \bullet A - \varepsilon\mu\, \partial\varphi/\partial t$. Here, A and φ are the vector and scalar potentials, with permeability and permittivity, ε and μ, respectively. The results are:

- relativistic covariance
- classical wave solutions
- elimination of inconsistency between the media-interface matching for φ and for Gauss' law
- independent determination of A and φ
- prediction of two new waves, one being a charge-fluctuation-driven scalar wave, having energy but not momentum
- a second longitudinal-electric wave with energy and momentum
- experimental suggestions.

Keywords: electrodynamics; electromagnetics; LEW; longitudinal electric wave; scalar wave.

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1 Introduction

Classical electromagnetism is central to all of physics, relating electric (***E***) and magnetic (***B***) fields with the dynamics of electrically charged matter. An ***E*** field is created by charges. The motion of charges creates an electrical current, which then influences the ***B*** and ***E*** fields.

Fundamental issues motivate a more complete, classical electro-dynamic theory. Maxwell's model is a coupled set of Partial-Differential Equations (PDEs) at a mathematical point (Maxwell, 1865). Equivalently, the forces depend only on the local field values. However, real experiments are performed over a finite time and spatial region, without any infinites. Divergences occur theoretically at a space-time point. Integral forms over a finite space-time region are a present alternative for Maxwell's equations (Jackson, 1962; Lorrain and Corson, 1970), but do not address another incompleteness, as discussed next.

A second motivation is that Maxwell's equations have an infinitude of arbitrary vector-field solutions. Solutions require a gauge assumption, such as $\nabla\bullet\boldsymbol{A}+\alpha\varepsilon\mu\,\partial\phi/\partial t=0$. One extreme is the Lorenz gauge ($\alpha=1$), implying that the effect of a charge source propagates at light speed, c. The Coulomb gauge ($\alpha=0$) yields electrostatics with ϕ propagation at infinite speed. Values of $0<\alpha<1$ are called the velocity gauge, and can be interpreted as ϕ propagation at a speed, $v>c$ (Jackson, 2002). Consequently, the gauge condition is related to (non)locality by its scalar-potential propagation speed.

An alternative assumes a dynamical variation in $\xi=-\nabla\bullet\boldsymbol{A}-\varepsilon\mu\,\partial\phi/\partial t$. Early work by Ohmura (1956) gave a time derivative of ξ in Gauss' law and a gradient of ξ in Ampere's law. Aharonov and Bohm (1963) added a term, $\xi^2/2\lambda$, to the classical electromagnetic Lagrangian for particle motion, where λ is a suitably adjusted constant. For $\lambda>0$, the theory predicts non-local sources for the current density, $\boldsymbol{J}\rightarrow\boldsymbol{J}-\nabla\xi/\mu$ and the charge density, $\rho\rightarrow\rho+\varepsilon\,\partial\xi/\partial t$. This theory also predicts non-conservation of charge, which is discussed below in more detail. As $\lambda\rightarrow 0$, the lowest excitation state of ξ approaches a delta function of $\nabla\bullet\boldsymbol{A}+\varepsilon\mu\,\partial\phi/\partial t$, allowing recovery of charge conservation and classical electrodynamics. For $\lambda=1$, the Aharonov-Bohm theory is identical to Ohmura (1956) and van Vlaenderen and Waser (2001). A term of the same form as $\xi^2/2\lambda$ can be added to the gauge-invariant Lagrangian density (Jackson and Okun, 2001); examples are the Feynman gauge ($\lambda=1$) and the Landau gauge ($\lambda=0$).

A third issue is that $\boldsymbol{A}$ and ϕ are uniquely and independently determined by the classical retarded Green's operators on $\boldsymbol{J}$ and ρ (Griffiths, 2007). However, the Lorenz gauge under classical electrodynamics implies that $\boldsymbol{A}$ and ϕ are not independent quantities, but rather are related by (Ribaric and Sustersic, 1990):

$$f(r,t)=f(r,s)-c^2\int_s^t \nabla\bullet A\,\mathrm{d}t. \tag{1}$$

The use of Ockham's Razor on this issue (Ribaric and Sustersic, 1990) favours the simpler hypothesis of $\boldsymbol{A}$ and ϕ as independent, which is explicitly allowed by this new theory. A test of this question involves simultaneous measurement of $\boldsymbol{A}$ and ϕ within an arbitrary gauge ($\boldsymbol{A}\rightarrow\boldsymbol{A}+\nabla\Lambda$ and $\phi\rightarrow\phi-\partial\Lambda/\partial t$). This theory also resolves a difference in matching conditions at the interface of two media, as obtained from the electric potential (ϕ) and from Gauss' law. This work does not address incompleteness for dielectric and magnetic materials, requiring the determination of ε and μ.

These issues suggest the need for a more complete electrodynamic theory, as discussed herein. Section 2 summarises relevant parts of classical electrodynamics. Section 3 elucidates the new theory with ξ as a dynamical quantity (Ohmura, 1956; Aharonov and Bohm, 1963; van Vlaenderen and Waser, 2001). Section 4 obtains a covariant wave equation and new forms for momentum and energy. Section 5 describes explicit wave solutions from the new theory. Section 6 discusses the new theory vs. present experimental tests of the Maxwell-Proca formulation for massive photons and measurement of longitudinal waves in free space (Giakos and Ishii, 1993). Section 7 provides the conclusions.

2 Classical electrodynamics

Electrodynamics can be modelled by Maxwell's equations (Jackson, 1962). Here, ***bold italic*** denotes a vector function of time and space. The electric current density and the electric charge density are denoted by $\boldsymbol{J}$ and ρ, respectively. The electric permittivity, ε, and magnetic permeability, μ, are assumed to be steady-state, homogeneous, isotropic, and not necessarily equal to their vacuum values. The ***B*** and ***E*** fields can be written in MKS units as (Lorrain and Corson, 1970):

$$\boldsymbol{B}=\nabla\times\boldsymbol{A}; \tag{2}$$

$$\boldsymbol{E}=-\partial\boldsymbol{A}/\partial t-\nabla\phi. \tag{3}$$

This paper focuses on wave-like solutions for the fields and potentials; other solutions (i.e., only of time, or only of space) are not discussed. The wave equations for ***B*** and ***E*** are (Lorrain and Corson, 1970):

$$\varepsilon\mu\,\partial^2\boldsymbol{B}/\partial t^2-\nabla^2\boldsymbol{B}=\mu\nabla\times\boldsymbol{J}; \tag{4}$$

$$\varepsilon\mu\,\partial^2\boldsymbol{E}/\partial t^2-\nabla^2\boldsymbol{E}=-\mu\,\partial\boldsymbol{J}/\partial t-\nabla\rho/\varepsilon. \tag{5}$$

The classical wave equations for ϕ and $\boldsymbol{A}$ have the form (Lorrain and Corson, 1970):

$$\varepsilon\mu\,\partial^2\phi/\partial t^2-\nabla^2\phi=\rho/\varepsilon; \tag{6}$$

$$\varepsilon\mu\,\partial^2\boldsymbol{A}/\partial t^2-\nabla^2\boldsymbol{A}=\mu\boldsymbol{J}. \tag{7}$$

The form, $\boldsymbol{B}=\nabla\times\boldsymbol{A}$, in equation (2) allows an arbitrary gauge function, Λ, via the vector identity, $\nabla\times\nabla\Lambda=0$ (Danese, 1965), implying that the vector potential is arbitrary under a transformation of $\boldsymbol{A}\rightarrow\boldsymbol{A}+\nabla\Lambda$, but ***B*** is unchanged (gauge invariant). ***E*** in equation (3) must also be invariant under this transformation, implying that $\phi \to \phi - \partial\Lambda/\partial t$. This infinitude of functional forms can be expressed as a choice of gauge transformation (Jackson and Okun, 2001; Jackson, 2002). One choice is the Lorenz gauge, $\xi = 0$, with the auxiliary condition, $\varepsilon\mu\, \partial^2\Lambda/\partial t^2 - \nabla^2\Lambda = 0$:

$$\xi = -\nabla \bullet \boldsymbol{A} - \varepsilon\mu\, \partial\phi/\partial t. \tag{8}$$

The use of $\xi = 0$ eliminates $\partial\xi/\partial t$ in the derivation of equation (6), and $-\nabla\xi$ in the derivation of equation (7).

3 The new theory

Aharonov and Bohm (1963) added a term, $\xi^2/2\lambda$, to the classical electromagnetic Lagrangian for particle motion. Here, λ is a suitably adjusted constant. $\boldsymbol{E}$ and ξ are then canonically conjugated to $\boldsymbol{A}$ and ϕ, respectively. As $\lambda \to 0$, they verify that the lowest excitation state of ξ approaches a delta function of ξ. As $\lambda \to 0$, they also recover charge conservation and the classical Maxwell's equations. They show that ξ as a *dynamical quantity* is indirectly equivalent to altering the assumptions on localisability of the charge-current interaction, resulting in charge non-conservation for $\lambda > 0$. Distributed (non-local) sources arise for $\boldsymbol{J} \to \boldsymbol{J} - \nabla\xi/\mu$ and for $\rho \to \rho + \varepsilon\partial\xi/\partial t$, also consistent with the present work. The present work uses the resultant equations of motion for $\lambda = 1$, which are identical to the work of Ohmura (1956), and van Vlaenderen and Waser (2001). Specifically, the new formulation adds one term in each inhomogeneous equation for $\boldsymbol{B}$ and $\boldsymbol{E}$ with $\xi \neq 0$, not unlike Maxwell's addition (Maxwell, 1865) of displacement current:

$$\nabla \times \boldsymbol{B} - \varepsilon\mu\, \partial\boldsymbol{E}/\partial t = \mu(\boldsymbol{J} - \nabla\xi/\mu); \tag{9}$$

$$\nabla \bullet \boldsymbol{E} = (\rho + \varepsilon\partial\xi/\partial t)/\varepsilon. \tag{10}$$

The homogeneous equations ($\nabla \bullet \boldsymbol{B} = 0$ and $\nabla \times \boldsymbol{E} + \partial\boldsymbol{B}/\partial t = 0$) are unchanged. The curl of equation (9) has the term, $\nabla \times \nabla\xi = 0$ (Danese, 1965), resulting in the classical $\boldsymbol{B}$-wave equation, equation (4). Moreover, ξ does not occur in the wave equation for $\boldsymbol{E}$, equation (5), because $\nabla\partial\xi/\partial t - \partial\nabla\xi/\partial t = 0$ from the gradient of equation (10). The wave equations for ϕ and $\boldsymbol{A}$ are also unchanged via cancellation of terms, $\partial\xi/\partial t$ and $\nabla\xi$, in equations (6) and (7), respectively. Consequently, the new theory preserves the classical wave equations for $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{E}$, and ϕ, as well as the classical expressions for the fields in terms of the potentials, equations (2) and (3). While the new theory is trivially consistent with the classical forms for $\xi = 0$, the key point of the new theory is ascribing *dynamical variability* to $\xi \neq 0$ (i.e., a scalar wave), rather than to focus on a particular choice of gauge. This approach is a significant revision of classical electrodynamics.

Matching conditions at the interface between two media are required to solve Maxwell's equations. This condition under classical electrodynamics for the normal component of $\boldsymbol{E}$ is (Lorrain and Corson, 1970):

$$\varepsilon_2 E_{n2} - \varepsilon_1 E_{n1} = \varepsilon_2(-\partial\boldsymbol{A}/\partial t - \nabla\phi)_{n2} - \varepsilon_1(-\partial\boldsymbol{A}/\partial t - \nabla\phi)_{n1} = \rho_A. \tag{11}$$

The subscript, 'n' denotes a component that is normal to the interface. The subscripts 1 and 2 indicate the corresponding value in media 1 or 2, respectively. The surface-charge density is ρ_A. The matching condition in equation (11) is unchanged by the Λ-gauge transformation, because $\boldsymbol{E}$ is unchanged.

The matching condition for equation (6) can be obtained by noting that $\nabla^2\phi = \nabla \bullet (\nabla\phi)$:

$$-\varepsilon_2(\nabla\phi)_{n2} + \varepsilon_1(\nabla\phi)_{n1} = \rho_A. \tag{12}$$

The difference between equations (11) and (12) is $\varepsilon_1(\partial\boldsymbol{A}/\partial t)_{n1} - \varepsilon_2(\partial\boldsymbol{A}/\partial t)_{n2} = 0$. This difference is not due to equation (12) being written in terms of the scalar potential, since equation (11) is Λ-gauge invariant in terms of the potential functions. The key assumption in the derivation of equation (12) is the choice of $\xi = 0$ for obtaining equation (6). Other gauges (e.g., $\nabla \bullet \boldsymbol{A} = 0$) are equivalent to $\xi = 0$ with different physical interpretations (Jackson and Okun, 2001; Jackson, 2002), so the issue is not the choice of gauge.

The matching condition for equation (6) under the new theory is equation (12) (The other matching conditions are unchanged). Equation (12) also can be obtained by substitution for $\boldsymbol{E}$ from equation (3) and $\xi \neq 0$ from equation (8) into equation (10). Consequently, the difference in matching condition is eliminated by this new theory. Moreover, equation (12) is a testable prediction of the theory, namely, that surface charge at an interface produces a discontinuity in the gradient of the scalar potential, rather than a discontinuity in the normal component of $\boldsymbol{E}$. An experiment with non-zero $\partial\boldsymbol{A}/\partial t$ should be able to test equation (11) vs. equation (12).

4 Waves, charge, momentum, and energy

A wave equation for ξ arises (van Vlaenderen and Waser, 2001; Arbab and Satti, 2009) from the divergence of equation (9), plus the time-derivative of equation (10):

$$\varepsilon\mu\, \partial^2\xi/\partial t^2 - \nabla^2\xi = -\mu[\partial\rho/\partial t + \nabla \bullet \boldsymbol{J}]. \tag{13}$$

Charge conservation corresponds to a zero right-hand side (RHS) for equation (13), which is an *instantaneous* equation. However, all real experiments are performed over a *finite time*, ΔT, corresponding to a time average. A long-time average gives $\partial\rho/\partial t + \nabla \bullet \boldsymbol{J} = 0$ on the RHS of equation (13), in accord with long-standing experimental evidence (Okun, 1989; Belli et al., 1999) for charge conservation, implying:

$$\langle \varepsilon\mu\, \partial^2\xi/\partial t^2 - \nabla^2\xi \rangle = 0,\ \Delta T >> \Delta t. \tag{14}$$

The zero RHS of equation (14) implies lossless ξ-wave propagation in the absence of a source or sink over long time scales, for a time average, $\langle\cdot\rangle$. Nevertheless, long-time-scale charge conservation does not preclude charge non-conservation over *short time scales*, and is a significant departure from the classical theory. In this context, 'short' means that $\Delta T \le \Delta t$, where Δt arises from the Heisenberg uncertainty relation, $\Delta E\, \Delta t \ge \hbar/2$; ΔE is the charged-quantum-fluctuation energy; and h is Planck's constant. Equation (13) can then be interpreted as charge non-conservation driving the ξ-wave, and vice versa. (This interpretation is not unlike energy fluctuations driving mass fluctuations in quantum electrodynamics, and vice versa.) Direct experimental confirmation of such quantum charge fluctuations will require observations, consistent with the Heisenberg uncertainty relation. For example, a bare electron has $\Delta E = m_e c^2 = 0.51$ MeV, corresponding to $\Delta t \sim 6 \times 10^{-22}$ seconds, which is too fast for direct observations at present. Still, this theory gives testable predictions for inference of ξ-waves over short time scales, as discussed below.

Regarding charge conservation, Okun (1989) reviewed the status of experimental tests (e.g., electron decay into two γ-rays, each at $m_e c^2/2$). A lower bound on the lifetime of an electron is $\tau_e > 10^{24}$ years, for long-time charge conservation; see Belli et al. (1999) and citations therein. Okun (1989) also discussed longitudinal photons (***E***-longitudinal waves, as discussed below), which would be emitted at the point in the Feynman diagram where charge conservation is violated (see also Heitler, 1984). Consequently, the longitudinal electric wave (LEW) is directly related to charge conservation, which is in turn tied to the ξ-wave under this theory. As discussed above, charge conservation over long-time scales is not inconsistent with short-time charge non-conservation under the Heisenberg uncertainty principle, which also allows non-conservation of energy and momentum.

Equation (13) can be derived directly, using the 4-vector form of equations (6) and (7) (Jackson, 1962):

$$\Box^2 \mathbf{A} = \mu \mathbf{J}. \tag{15}$$

Here, the 4-vector current density is $\mathbf{J} = (\boldsymbol{J}, ic\rho)$. The 4-vector potential is $\mathbf{A} = (\boldsymbol{A}, i\phi/c)$, using $i = (-1)^{1/2}$, $\Box = (\nabla, \partial/\partial ict)$, and $c = (\varepsilon\mu)^{-1/2}$ in the local medium (not necessarily vacuum). As discussed in Section 3, equation (15) arises from this more complete theory without assuming a gauge condition. The 4-divergence of equation (15) is $\Box \bullet (\Box^2 \mathbf{A}) = \mu \Box \bullet \mathbf{J}$, in which the time derivatives commute with one another and with the spatial derivatives. Also, the Laplacian commutes with the divergence of $\boldsymbol{A}$, $\nabla \bullet (\nabla^2 \boldsymbol{A}) = \nabla^2 (\nabla \bullet \boldsymbol{A})$, because $\nabla \bullet (\nabla \times \boldsymbol{B}) = 0$ and $\boldsymbol{B} = \nabla \times \boldsymbol{A}$. Thus, the D'Alembertian commutes with the 4-divergence to yield:

$$\Box^2 (-\Box \bullet \mathbf{A}) = -\mu \Box \bullet \mathbf{J}. \tag{16}$$

The RHS of equation (16) can be written (Jackson, 1962) as the RHS of equation (13), $-\mu \Box \bullet \mathbf{J} = -\mu(\nabla \bullet \boldsymbol{J} + \partial\rho/\partial t)$. Moreover, $-\xi$ is the divergence of the 4-vector potential (Jackson, 1962). Consequently, equation (16) is an alternative form for equation (13). Under classical electrodynamics, the right- and left-hand sides of equation (16) are independently zero. Equation (13) is equivalent to equation (16) under the new theory, as a 4-vector covariant form that is consistent with relativistic invariance (Jackson, 1962). This derivation elucidates the Aharonov-Bohm (1963) statement about classical charge conservation, corresponding to a delta function of $\nabla \bullet \boldsymbol{A} + \varepsilon\mu\, \partial\phi/\partial t$ ($\xi = 0$) for the wave function for the lowest excitation state. This derivation also provides insight into charge non-conservation ($\Box \bullet \mathbf{J} \neq 0$), arising from the dynamical variability in $\xi = -\Box \bullet \mathbf{A}$ under this more complete theory. The ξ-wave boundary condition is:

$$(\nabla\xi)_{n2}/\mu_2 - (\nabla\xi)_{n1}/\mu_1 = [\partial\rho_A/\partial t + \nabla \bullet \boldsymbol{J}_A]. \tag{17}$$

This condition corresponds to a discontinuity in $\nabla\xi$ from charge fluctuations at an interface.

The resultant form for momentum balance under the new theory is (van Vlaenderen and Waser, 2001):

$$\varepsilon\mu\, (\partial \boldsymbol{P}/\partial t) + (\rho + \varepsilon\partial\xi/\partial t)\boldsymbol{E} + (\boldsymbol{J} - \nabla\xi/\mu) \times \boldsymbol{B} = \nabla \bullet \underline{\boldsymbol{T}}. \tag{18}$$

The form for energy conservation under the new theory becomes (van Vlaenderen and Waser, 2001):

$$\partial u/\partial t + \nabla \bullet \boldsymbol{P} + (\boldsymbol{J} - \nabla\xi/\mu) \bullet \boldsymbol{E} = 0, \text{ with } u = \tfrac{1}{2}\,(\varepsilon \boldsymbol{E}^2 + \boldsymbol{B}^2/\mu). \tag{19}$$

The Poynting vector is $\boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{B}/\mu$; the Maxwell stress tensor is $\underline{\boldsymbol{T}} = \varepsilon[\boldsymbol{EE} - \tfrac{1}{2}\boldsymbol{I}\boldsymbol{E}^2] + [\boldsymbol{BB} - \tfrac{1}{2}\boldsymbol{I}\boldsymbol{B}^2]/\mu$, from the classical definitions (Jackson, 1962). An alternative form is (van Vlaenderen and Waser, 2001):

$$\partial v/\partial t + \nabla \bullet \boldsymbol{Q} + \boldsymbol{J} \bullet \boldsymbol{E} + \rho\xi/\varepsilon\mu = 0, \text{ with}$$
$$v = \tfrac{1}{2}\,(\varepsilon \boldsymbol{E}^2 + \boldsymbol{B}^2/\mu + \xi^2/\mu);\ \boldsymbol{Q} = \boldsymbol{E} \times \boldsymbol{B}/\mu - \xi\boldsymbol{E}/\mu. \tag{19′}$$

The new form for the Lorentz force from equation (18) then is (van Vlaenderen and Waser, 2001):

$$\boldsymbol{F} = (\rho + \varepsilon\partial\xi/\partial t)\boldsymbol{E} + (\boldsymbol{J} - \nabla\xi/\mu) \times \boldsymbol{B}. \tag{20}$$

Equations (18)–(20) have the current density, $\boldsymbol{J}$, replaced by $(\boldsymbol{J} - \nabla\xi/\mu)$; ρ is replaced by the term, $(\rho + \varepsilon\partial\xi/\partial t)$. These replacements arise from equations (9) and (10), which have the same replacements. Validation of equation (20) requires measurement of $\varepsilon(\partial\xi/\partial t)\ \boldsymbol{E}$ and $(\nabla\xi/\mu) \times \boldsymbol{B}$. The third wave solution (below) shows that ξ can occur together with a longitudinal ***E***-wave. Consequently, this theory predicts that long-time-averaged products, $<(\partial\xi/\partial t)\boldsymbol{E}>$ and $<\nabla\xi \times \boldsymbol{B}>$, in the generalised force of equation (20) will be experimentally observable, due to correlations between the ξ- and electromagnetic waves, even if the short-time variations in ξ, $\boldsymbol{B}$, and $\boldsymbol{E}$ are too fast to observe directly.

5 New wave solutions

This section addresses explicit wave solutions for $\boldsymbol{B} = \boldsymbol{E} = 0$. The resultant forms are:

$$-\partial\xi/\partial t = \rho/\varepsilon, \text{ for } \boldsymbol{E} = 0 \text{ in equation (10);} \tag{21}$$

$$\nabla\xi = \mu\boldsymbol{J}, \text{ for } \boldsymbol{B} = \boldsymbol{E} = 0 \text{ in equation (9);} \tag{22}$$

$$\nabla \times \boldsymbol{J} = 0, \text{ for } \boldsymbol{B} = 0 \text{ in equation (4);} \tag{23}$$

$$\partial\boldsymbol{J}/\partial t + \nabla\rho/\mu\varepsilon = 0, \text{ for } \boldsymbol{E} = 0 \text{ in equation (5).} \tag{24}$$

Equations (21)–(24) have no wave-like solutions for ρ = constant or $\boldsymbol{J}$ = constant. Rather, these conditions are for electrostatics and magnetostatics, respectively. Equations (21) and (22) are equivalent to equations (23) and (24). Note that equation (22), $\nabla\xi = \mu\boldsymbol{J}$, implies that the line integral, $\int \boldsymbol{J} \bullet d\boldsymbol{\ell} = \int \nabla\xi \bullet d\boldsymbol{\ell}/\mu$, is independent of the path, and thus is zero around a closed path, corresponding to no circulating currents for creation of the ξ-wave. Rather, ξ arises from charge fluctuations in equation (13). The condition, $\boldsymbol{B} = 0$, implies that $\nabla \times \boldsymbol{A} = 0$ from equation (2), or $\boldsymbol{A} = \nabla\chi$ for an arbitrary scalar function, χ. The resultant equation is:

$$\Box^4 \chi = \mu[\partial\rho/\partial t + \nabla \bullet \boldsymbol{J}],\ \boldsymbol{A} = \nabla\chi \text{ and } \phi = -\partial\chi/\partial t. \tag{25}$$

Here, the D'Alembertian operator is: $\Box^2 = \nabla^2 - \partial^2/\partial c^2 t^2$, where $c = (\varepsilon\mu)^{-1/2}$ in the local medium (not necessarily vacuum), as before. Charged fluctuations drive this fourth-order equation. The momentum, $\boldsymbol{Q}$, in equation (19′) yields zero for $\boldsymbol{E} = \boldsymbol{B} = 0$, along with an energy density of the form, $v\,(\boldsymbol{E} = \boldsymbol{B} = 0) = \xi^2/2\mu$. Thus, ξ-waves have energy, but not momentum. The explicit prediction of wave-like phenomena (ξ-waves) with energy, but not momentum, is unprecedented, but is not unlike charged, quantum particle-antiparticle fluctuations having energy, but no net momentum.

A second special case for $\boldsymbol{E} = 0$ gives $\boldsymbol{A} = \nabla\chi$ and $\phi = -\partial\chi/\partial t$. However, substitution of $\boldsymbol{A} = \nabla\chi$ into equation (2) yields $\boldsymbol{B} = 0$. Consequently, $\boldsymbol{E} = 0$ implies $\boldsymbol{B} = 0$, which is the same as the previous case. Thus, magnetic wave-like solutions cannot occur for $\boldsymbol{E} = 0$ with the ξ-wave.

A third case for $\boldsymbol{B} = 0$ also gives $\boldsymbol{A} = \nabla\chi$. Equation (4) simplifies to $\nabla\times\boldsymbol{J} = 0$, implying that $\boldsymbol{J} = \nabla\kappa$ for an arbitrary scalar function, κ. The resultant wave equation is:

$$\varepsilon\mu\,\partial^2\chi/\partial t^2 - \nabla^2\chi = \mu\kappa, \text{ with } \boldsymbol{J} = \nabla\kappa \text{ and } \boldsymbol{A} = \nabla\chi. \tag{26}$$

Two scalar-wave equations are involved: equation (26) for χ and equation (6) for ϕ. The field is $\boldsymbol{E} = -\nabla(\phi + \partial\chi/\partial t)$, which has momentum, $\boldsymbol{Q}(\boldsymbol{B} = 0) = -\xi\boldsymbol{E}/\mu$, from equation (19′), and an energy density is $v(\boldsymbol{B} = 0) = \tfrac{1}{2}\,(\varepsilon\boldsymbol{E}^2 + \xi^2/\mu)$. This solution has a ξ-wave, together with a longitudinal $\boldsymbol{E}$-wave, as indicated by the term, $\xi\boldsymbol{E}$, in equation (19′).

These new waves directly drive currents via the gradient of a scalar field, $\boldsymbol{J} = \nabla\xi/\mu$, from equation (25) for $\boldsymbol{B} = \boldsymbol{E} = 0$, and $\boldsymbol{J} = \nabla\kappa$ from equation (26) for $\boldsymbol{B} = 0$. Consequently, electrical-conductivity losses do not occur for longitudinal and ξ-waves. By contrast, transverse waves under classical electrodynamics drive currents in proportion to the electric field (Jackson, 1962; Corson and Lorrain, 1970), $\boldsymbol{J} = \sigma\boldsymbol{E}$, with a corresponding 'skin depth' and longitudinal decay time ($\varepsilon/\sigma \sim 10^{-17}$ s) for typical conductive media.

The nature of the ξ-wave can be elucidated further by the fundamental theorem of vector calculus, which states that a sufficiently smooth, decaying vector field, $\boldsymbol{R}$, has a unique decomposition, as the sum of curl-free and divergence-free components (Danese, 1965):

$$\boldsymbol{R} = \nabla u + \nabla\times\boldsymbol{W}. \tag{27}$$

If $\nabla\bullet\boldsymbol{R} = 0$, then $\boldsymbol{R} = \nabla\times\boldsymbol{W}$ from the identity (Danese, 1965), $\nabla\bullet\nabla\times\boldsymbol{W} = 0$. Here, $\boldsymbol{W}$ is the 'vector' potential for this 'solenoidal' or 'divergence-free' case. A solenoidal current, $\nabla\times\boldsymbol{J} \neq 0$, drives these 'transverse' electromagnetic waves, which are well-known in classical electrodynamics (Jackson, 1962; Griffiths, 2007). If $\nabla\times\boldsymbol{R} = 0$, then equation (27) has the form, $\boldsymbol{R} = \nabla u$, via the identity (Danese, 1965), $\nabla\times\nabla u = 0$. Here, u is the 'scalar' potential for this 'irrotational' or 'curl-free' case with a current, $\boldsymbol{J} = \nabla u$, exciting the waves in equations (25) and (26). This gradient-driven current is distinct from the solenoidal current that drives transverse waves.

6 Discussion

How does this theory compare to tests of Gauss' law, Ampere's law, and charge conservation? Tests of Gauss' and Ampere's law are based on falsifiability tests for massive photons under the Maxwell-Proca theory, which has an additional term on the RHS of Gauss' law ($-\mu_\gamma^2\phi$) and an additional term on the RHS of Ampere's law ($-\mu_\gamma^2\boldsymbol{A}$). The photon mass is μ_γ, with present experimental limits at a level of $\mu_\gamma < 10^{-51}$ g; see Luo et al. (2003) and Goldhaber and Nieto (2010). Such tests cannot be used to test the new theory, which is incompatible with the Maxwell-Proca formulation. As discussed above, the long-time lower bound on the lifetime for single electron decay, $\tau_e > 10^{24}$ years (Belli et al., 1999) does not preclude short-time non-conservation of charge. No short-time tests of charge conservation are known.

Classical theory predicts no LEW in free space. One classical counter-example arises from a stellar explosion, which ejects concentric, spherical, radially expanding shells of fast electrons (outer shell) and slower-moving positive-ions (inner shell). These charged shells form a spherical capacitor with a radial $\boldsymbol{E}$-field that is parallel to the shell's outward radial motion. Spherical symmetry implies that the $\boldsymbol{E}$-field varies as $1/R^2$, with R = radial distance from the centre. If the ejecta are not spherically symmetric (e.g., dipole shells from the Crab supernova), an $\boldsymbol{E}$-field component still occurs in the outgoing direction. The temporal and radial variations in this propagating $\boldsymbol{E}$-field form a LEW. A second example is a nuclear explosion in vacuum that produces an LEW for the same reason (Monstein and Wesley, 2002). A third example is a flat-plate capacitor, which has a longitudinal $\boldsymbol{E}$-field that changes in time and space between the conductive plates during (dis)charging, even with edge effects; this example is also valid for other geometries. The temporal variations in charge separation drive the boundary conditions in each of these examples, which can occur (in principle) at infinity, making the propagation medium indistinguishable from free space. (Other classical situations also yield longitudinal electric fields, such as wave propagation at the interface between two dielectric materials, and TE/TM modes in waveguides.) Experimental validation is, therefore, vital.

Giakos and Ishii (1993) measured longitudinal waves in air over many wavelengths from the open end of a rectangular waveguide at 12.25 GHz (free-space wavelength of 2.45 cm), as shown in Figure 1. The entire experiment was elevated 82.5 cm above a wooden tabletop at two different indoor sites to minimise any ground waves and reflections. The $\boldsymbol{E}$-field sensor was a 3.5 cm long dipolar antenna of 0.0123 cm thick copper wire, oriented along the Z axis. The non-decaying, free-space, longitudinal electromagnetic wave has interference oscillations, as shown in Figure 2. On the other hand, the $\boldsymbol{B}$-wave sensor was a 3.5 cm, square loop antenna of 0.0123 cm thick copper wire, oriented with the axis of the loop along the Z axis. A non-decaying longitudinal $\boldsymbol{B}$-wave was observed, as shown in Figure 3. This result follows from classical electrodynamics in conversion of the wave-guide modes to low-loss, free-space propagation; see Giakos and Ishii (1993) and references therein. The observed longitudinal components do not decay according to the $1/R^2$ or $1/R^3$ with distance, but at a much slower rate. The interference oscillations probably arose from the coupling of hybrid TE/TM modes, due to the non-uniformity of the surface impedance of the antennas. The non-evanescent, longitudinal component of the electric wave in free space is not inconsistent with this new theory. However, this experiment needs to be repeated with much more care for validation of the new theory. For example, the cause of the interference oscillations cannot be verified, because the excitation levels of the TE/TM mode(s) were not specified. The antennas had no electromagnetic shielding to avoid interference among the various field components. Vertical (and several different horizontal) waveguide orientations should produce the same results.

Figure 1 Schematic diagram of the experimental arrangement (E=electric dipole; M=magnetic dipole}, showing the angle of observation and orientation of the dipoles. Different orientations are indicated as solid lines for the longitudinal field, and dashed lines for the transverse field (Giakos and Ishii, 1993) (see online version for colours)

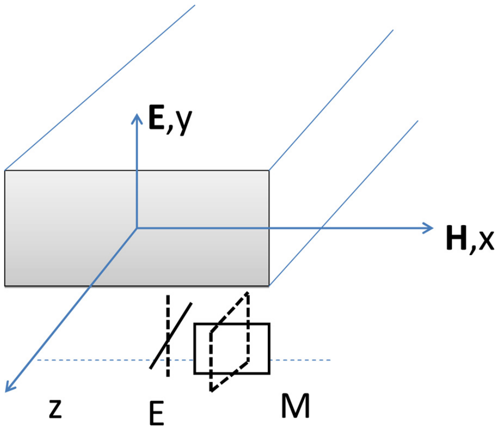


Figure 2 Predominantly longitudinal electric field vs. parallel shifting distance of the detector (rectangular dipole antenna) in air with a line-of-sight distance of 75.5 cm at an operating frequency of 12.25 GHz (Giakos and Ishii, 1993)

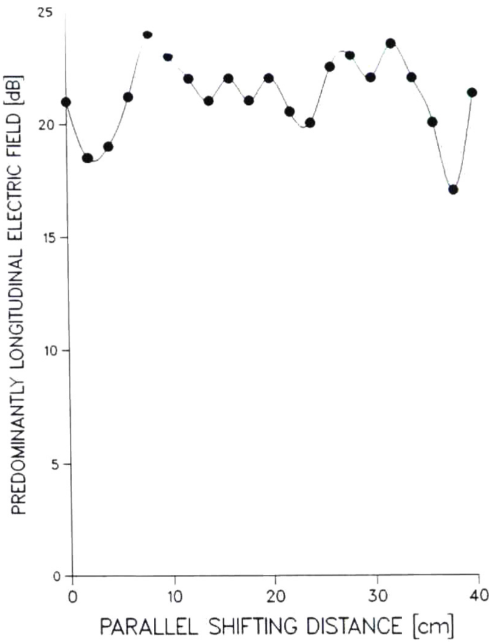
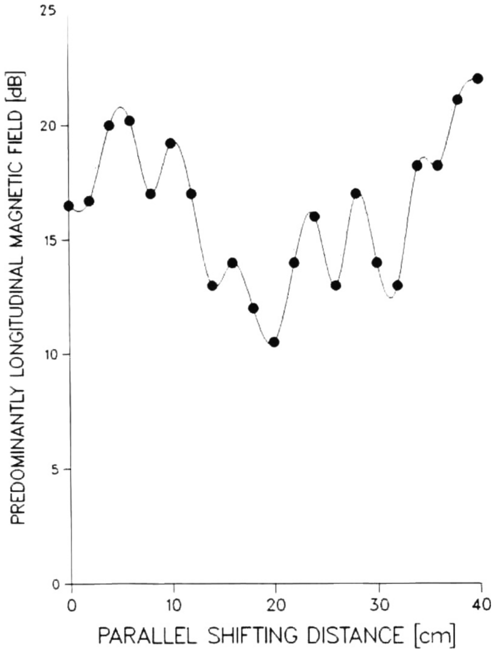


Figure 3 Predominantly longitudinal magnetic field vs. parallel shifting distance of the detector (rectangular loop antenna) in air with a line-of-sight distance of 75.5 cm at an operating frequency of 12.25 GHz (Giakos and Ishii, 1993)



Classically, radial oscillations of a charged sphere produce no electromagnetic radiation (Panofsky and Phillips, 1969), because spherical symmetry implies that $-\partial \boldsymbol{B}/\partial t = \nabla \times \boldsymbol{E} = 0$, i.e., no magnetic wave. Thus, the Poynting vector, $\boldsymbol{E} \times \boldsymbol{B}/\mu$, is zero, yielding no classical electromagnetic radiation. Under this new theory, a charged, oscillating sphere produces both a LEW and a scalar wave. An experiment by Ignatiev and Leus (2001) used a radially oscillating (120 kHz) charged sphere to generate a longitudinal electric wave, for which the propagation speed was measured at 1.12c. However, details are unspecified (e.g., laboratory setting, antenna geometry, magnetic-solenoid geometry for excitation of oscillations in the sphere, sensor). Moreover, measurements were in the near-field regime (wavelength=2.5 km), for which ground reflections and interferences are important. This experiment needs replication to validate the new theory. The reciprocity theorem guarantees transmission and detection by the same geometry.

A computational electromagnetic model can be modified for the new theory, with simulations to guide the above experiments. For example, is a TM_{01} mode (with a large $\boldsymbol{E}_z$) best to maximise the free-space amplitude of $\boldsymbol{E}_z$ in the LEW waveguide experiment? Will coupling to other TE/TM modes in that experiment produce the interference oscillations, as discussed above? An important question for the oscillating-charged-sphere experiment is whether monopole oscillations will produce analogous interference oscillations when coupled with multi-pole oscillations in the sphere. Detailed (graphical) results of such analyses are beyond the scope of the present work.

The present work differs from 'generalised Maxwell's equations', which include magnetic charge and currents,

Table 1 Summary of predictions

Classical prediction	*New theory*	*Testable by measurement of ...*
$f(r,t) = f(r,s) - c^2\int_s^t \nabla \bullet A\mathrm{d}t$	$\boldsymbol{A}$ and ϕ as independent quantities	Correlation between $\boldsymbol{A}$ and ϕ
$\varepsilon_2 E_{n2} - \varepsilon_1 E_{n1} = \rho_A$	$-\varepsilon_2(\nabla\phi)_{n2} + \varepsilon_1(\nabla\phi)_{n1} = \rho_A$	$\varepsilon_1(\partial\boldsymbol{A}/\partial t)_{n1} - \varepsilon_2(\partial\boldsymbol{A}/\partial t)_{n2} \neq 0$?
Wave equations for $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{E}$, ϕ	Wave equations for $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{E}$, ϕ	Waves
$\xi = 0$	$\varepsilon\mu\, \partial^2\xi/\partial t^2 - \nabla^2\xi = -\mu[\partial\rho/\partial t + \nabla \bullet \boldsymbol{J}]$	ξ-wave: energy/no momentum
Evanescent $\boldsymbol{E_Z}$-wave	Free space $\boldsymbol{E_Z}$-wave	$\boldsymbol{E_Z}$-wave: energy + momentum
Charge conservation	Charge conservation at long-time scale	short-time charge conservation
$\boldsymbol{F} = \rho\boldsymbol{E} + \boldsymbol{J}\times\boldsymbol{B}$	$\boldsymbol{F} = (\rho + \varepsilon\partial\xi/\partial t)\boldsymbol{E} + (\boldsymbol{J} - \nabla\xi/\mu)\times\boldsymbol{B}$	$\varepsilon\boldsymbol{E}\,\partial\xi/\partial t$ and $\nabla\xi\times\boldsymbol{B}/\mu$

beginning with the early work of Dirac (1931). Other recent theoretical work formulates four-potential solutions for $\boldsymbol{B}$ and $\boldsymbol{E}$ (Dubovik et al., 2000). More recent work by Nahara (arXiv:1006.5026v5 2010) writes the four-potentials in terms of two scalar potentials (electroscalar and magnetoscalar), and reduces to this new theory for no magnetic charge and no magnetic current.

7 Conclusions

This paper examines the implications of a new electrodynamic theory (Ohmura, 1956; Aharonov and Bohm, 1963; van Vlaenderen and Waser, 2001), with dynamical variation in $\xi = -\nabla \bullet \boldsymbol{A} - \varepsilon\mu\, \partial\phi/\partial t$. This new theory adds one new term to both Gauss' law ($\partial\xi/\partial t$) and Ampere's law ($-\nabla\xi$). The new theory preserves the classical wave equations, as well as the classical fields in terms of the potentials. The new theory resolves a difference in the matching condition for Gauss' law and the ϕ-wave equation, making $\boldsymbol{A}$ and ϕ explicitly independent. The new theory predicts a non-evanescent longitudinal-electric wave. Dynamical variability in ξ is interpreted as a scalar wave that is driven by charge fluctuations on fast time scales; charge conservation is preserved for long times. (This interpretation is not unlike energy fluctuations driving mass fluctuations in quantum electrodynamics, and vice versa. The precedent for the latter is supportive of the former.) The new theory predicts modified forms for the Lorentz force, the Poynting vector, and electromagnetic energy density.

Table 1 summarises the predictions of this new theory. Some of these predictions are identical to classical electrodynamics (e.g., wave equations for $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{E}, \phi$; charge conservation on long time scales). Other predictions are a departure from classical theory. One change involves non-local source terms for current density, $\boldsymbol{J} \rightarrow \boldsymbol{J} - \nabla\xi/\mu$, and charge density, $\rho \rightarrow \rho + \varepsilon\partial\xi/\partial t$, which appear in the force and energy balance equations. Another difference is non-conservation of charge on short time scales. Experimental tests of these predictions are needed (Okun, 1989; Belli et al., 1999) and are a subject of future work.

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References

Aharonov, V. and Bohm, D. (1963) 'Further discussion of the role of electromagnetic potentials in the quantum theory', *Phys. Rev.*, Vol. 130, p.1625.

Arbab, A.I. and Satti, Z.A. (2009) 'On the generalized Maxwell equations and their prediction of the electroscalar wave', *Prog. Phys.*, Vol. 2, No. 8, pp.8–13.

Belli, P. *et al.* (1999) 'Charge non-conservation restrictions from the nuclear levels excitation of ^{129}Xe induced by the electron's decay on the atomic shell', *Phys. Lett. B*, Vol. 465, pp.315–322.

Danese, A.E. (1965) *Advanced Calculus*, Allyn & Bacon, Inc., Boston.

Dirac, P.A.M. (1931) 'Quantised singularities in the electromagnetic field', *Proc. R. Soc. London, Ser. A*, Vol. 133, p.60.

Dubovik, V.M., Martsenyuk, M.A. and Saha, B. (2000) 'Material equations for the electromagnetism with toroidal polarizations', *Phys. Rev. E*, Vol. 61, p.7087.

Giakos, G.C. and Ishii, T.K. (1993) 'Detection of longitudinal electromagnetic fields in air', *Microwave Optical Techn. Lett.*, Vol. 6, pp.283–287.

Goldhaber, A.S. and Nieto, M.M. (2010) 'Photon and graviton mass limits', *Rev. Mod. Phys.*, Vol. 82, pp.939–979.

Griffiths, D.J. (2007) *Introduction to Classical Electrodynamics*, Prentice-Hall of India, New Delhi.

Heitler, W. (1984) *The Quantum Theory of Radiation*, Dover Publ., NY.

Ignatiev, G.F. and Leus, V.A. 2001 'On a superluminal transmission at the phase velocities', in Chubykalo, A.E., Pope, V. and Smirnov-Rueda, R. (Eds.): *Instantaneous Action at a Distance in Modern Physics: 'Pro' and 'Contra'*, Nova Science Publ., NY, pp.203–207.

Jackson, J.D. (1962) *Classical Electrodynamics*, John Wiley & Sons, Inc., New York.

Jackson, J.D. (2002) 'From Lorenz to Coulomb and other explicit gauge transformations', *Am. J. Phys.*, Vol. 70, p.917.

Jackson, J.D. and Okun, L.B. (2001) 'Historical roots of gauge invariance', *Am. J. Phys.*, Vol. 73, p.663.

Lorrain, P. and Corson, D.R. (1970) *Electromagnetic Fields and Waves*, W.H. Freeman & Co, San Francisco.

Luo, J., Tu, L-C., Hu, Z-K. and Luan, E-J. (2003) 'New experimental limit on the photon rest mass with a rotating torsion balance', *Phys. Rev. Lett.*, Vol. 90, Paper #081801, pp.081801-1–081801-4.

Maxwell, J.C. (1865) 'A dynamical theory of the electromagnetic field', *Phil. Trans. Royal Soc.*, London, Vol. 155, p.459.

Monstein, C. and Wesley, J.P. (2002) 'Observation of scalar longitudinal electrodynamic waves', *Europhys. Lett.*, Vol. 59, pp.514–520.

Ohmura, T. (1956) 'A new formulation on the electromagnetic field', *Prog. Theor. Phys.*, Vol. 16, pp.684–685.

Okun, L.B. (1989) 'Tests of electric charge conservation and the Pauli principle', *Sov. Phys. Usp.*, Vol. 32, pp.543–547.

Panofsky, W.K.H. and Phillips, M. (1969) *Classical Electricity and Magnetism*, 2nd ed., Addison-Wesley Publ., Reading, MA.

Ribaric, M. and Sustersic, L. (1990) *Conservation Laws and Open Questions of Classical Electrodynamics*, World Scientific, Singapore.

van Vlaenderen, K.J. and Waser, A. (2001) 'Generalisation of classical electrodynamics to admit a scalar field and longitudinal waves', *Hadronic J.*, Vol. 24, pp.609–628.